Σの公式とΣの計算全パターン。Σの考え方のすべて　解答

(1) $\displaystyle\sum_{k=1}^{n}\frac{2^k(1-k)}{k(k+1)}$　　(2) $\displaystyle\sum_{k=1}^{n}k!\cdot k$　　(3) $\displaystyle\sum_{k=1}^{n}\frac{k}{(k+1)!}$

解答

(1) $\dfrac{1-k}{k(k+1)}=\dfrac{1}{k}-\dfrac{2}{k+1}$ に着目して，$\dfrac{2^k(1-k)}{k(k+1)}=\dfrac{2^k}{k}-\dfrac{2^{k+1}}{k+1}$

これより，

$$\sum_{k=1}^{n}\frac{2^k(1-k)}{k(k+1)}$$

$$=\sum_{k=1}^{n}\left(\frac{2^k}{k}-\frac{2^{k+1}}{k+1}\right)$$

$$=\left\{\left(\frac{2}{1}-\frac{2^2}{2}\right)+\left(\frac{2^2}{2}-\frac{2^3}{3}\right)+\cdots+\left(\frac{2^n}{n}-\frac{2^{n+1}}{n+1}\right)\right\}$$

$$=2-\frac{2^{n+1}}{n+1}\quad\cdots\text{（答）}$$

(2) $k!\cdot k=k!\cdot\{(k+1)-1\}=(k+1)!-k!$

これより，

$$\sum_{k=1}^{n}k!\cdot k$$

$$=\sum_{k=1}^{n}\{(k+1)!-k!\}$$

$$=\{(2!-1!)+(3!-2!)+\cdots+((n+1)!-n!)\}$$

$$=(n+1)!-1\quad\cdots\text{（答）}$$

(3) $\dfrac{k}{(k+1)!}=\dfrac{(k+1)-1}{(k+1)!}=\dfrac{1}{k!}-\dfrac{1}{(k+1)!}$

これより，

$$\sum_{k=1}^{n}\frac{k}{(k+1)!}$$

$$=\sum_{k=1}^{n}\left\{\frac{1}{k!}-\frac{1}{(k+1)!}\right\}$$

$$=\left\{\left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\cdots+\left(\frac{1}{n!}-\frac{1}{(n+1)!}\right)\right\}$$

$$=1-\frac{1}{(n+1)!}\quad\cdots\text{（答）}$$